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# A Stocking Guide for European Larch in Eastern North America

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**ABSTRACT:** *European larch is a fast-growing exotic species that shows promise in alleviating projected fiber supply shortages with numerous plantations being established in the Lake States, the Northeast, and Canada during the last two decades. Prescribing thinning regimes for these plantations would be facilitated through the use of a stocking chart. Few older plantations exist in North America, however, thus fitting equations to construct an A-line that requires data from a large number of stands is not possible. We introduce a new approach to constructing an A-line by using the stand density index that in essence is a modification of the traditional tree area ratio approach. We then invoke the self-thinning rule to produce a reference line for Maximum Average Density and an equation for open-grown trees to produce a reference line for Full Site Occupancy in the construction a stocking chart for field use. We also provide yield estimates by diameter class at maximum and minimum stocking levels. North. J. Appl. For. 20(1):34–38.*

**Key Words:** Basal area, forest plantations, productivity, self thinning, silviculture, spacing.

For over two and a half centuries, European larch (*Larix decidua* Miller) has been recognized as a fast growing exotic species exhibiting impressive volume growth in North America (Cook 1969). Impressive early height and volume growth have been observed in numerous provenance tests and plantation trials throughout the Northeast, Lake States, and eastern Canada (Aird and Stone 1955, Carter et al. 1981, Park and Fowler 1983, Einsphar et al. 1984, Carter and Selin 1987, Mroz et al. 1988). Once plantations are established, periodic thinning may be required to realize the optimal production of high quality fiber. Stocking guides are useful decision tools for the scheduling of thinnings based on stand density and average stand diameter.

The stocking guide concept (Gingrich 1967) has undergone numerous modifications and improvements (Ernst and Knapp 1985, Seymour and Smith 1987, Halligan and Nyland 1999). The purpose of this article is to introduce an

alternative approach for the development of a stocking guide to aid managers in prescribing thinning regimes for European larch.

## Materials and Methods

### Study Area

Fixed area, temporary sample plots were established in all known European larch plantations planted in Maine prior to 1981 during the summers of 1989 and 1990 as part of a comprehensive growth and yield study (Gilmore et al. 1993, 1994, Gilmore and Briggs 1996). Plantations were located in three climatic zones (Briggs and Lemin 1992). Elevations ranged from 115 to 1000 ft above sea level. Soils were representative of Spodosols, Inceptisols, and Entisols, of glacial or glacial-fluvial origin, acidic, sand to silt loam in texture, and characterized by a frigid temperature regime. Latitude ranged from 44°56' to 45°16'N and longitude ranged from 68°39' to 70°35'E.

### Data Collection

Plot locations were selected to exclude areas disturbed from harvesting, to avoid areas having excessive mortality, and to minimize interspecific competition. Thirty-one plots were located in 12 plantations in central Maine, of which 16 plots in 7 plantations had up to 25% tree mortality that was assumed to be from self-thinning. Initial planting densities of the older plantations are unknown, but a narrow range of planting densities (8 × 8 ft to 11 × 11 ft) was encountered in

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the younger plantations. Mortality from insect and disease vectors was not observed. Diameters at breast height (dbh measured at 4.5 ft) were measured (to the nearest 0.1 in.) on all trees, and height measurements (to the nearest 1 ft) were measured on three to five trees representative of each crown class. Site indices (index age of 20 yr at bh) were obtained from stem analysis data or estimated from early height growth information (Gilmore et al. 1993). Data from 13 circular 0.05 ac, and 3 circular 0.02 ac plots were converted to a common per ac basis for analyses (Table 1).

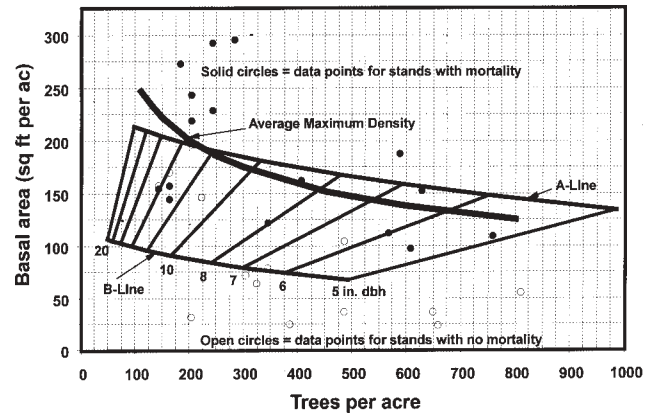
## Data Analysis

### Determination of a Minimum Stocking Level

Crown area was predicted for each diameter class using the projected crown radius prediction equations developed for open-grown larch in Minnesota (Gilmore 2001). Basal area and number of trees required for complete crown closure (assuming that crowns are sufficiently distorted to fill all voids in the canopy) were determined for each diameter class.

### Determination of Average Maximum Density

Measures used to quantify stocking and competition are the tree area ratio (TAR, Chisman and Schumacher 1940) and the crown competition factor (CCF, Krajicek et al. 1961). TAR is the ratio of crown area to plot area, but when originally presented, TAR was not calculated using direct measures of crown area. CCF is calculated as the sums of maximum crown projection area (calculated using crown radius equations for open-grown trees) per plot area divided by plot area. TAR would be equal to 1 and CCF would be equal to 100 on a plot where sufficient aboveground growing space was present to allow all trees to develop an open-grown crown size but still have complete crown closure. Calculations of TAR and CCF require a large number of sample points, and once constructed, these equations sometimes exhibit irrational behavior in the construction of an A-line, necessitating the application of a smoothing technique to reflect biological reality (Halligan and Nyland 1999). We did not have enough sample points to calculate TAR or CCF using the analytical techniques of Chisman and Schumacher (1940) and Krajicek et al. (1961). Recent approaches to construct stocking charts suggest replacement of the historical A-line with an Average Maximum Density reference level (Ernst and Knapp 1985, Halligan and Nyland 1999). Pragmatically, the A-line and the Average Maximum Density reference line would serve the same purpose. Historically, A-line construction did not involve the measurement of tree crowns. Assuming that tree mortality in the understory is an indicator that self-thinning is occurring, and further assuming that self-thinning is an indicator of maximum



**Figure 1. Stocking chart for European larch comparing the line of Average Maximum Density calculated using the self-thinning rule with the A-line calculated using a modification of the traditional approach based on the tree area ratio. Data from stands included in this study are provided for reference (solid circles = data from stands with tree mortality; open circles = data from stands without tree mortality).**

stocking, only data from plots having larch mortality were used in the development of an Average Maximum Density reference level.

Our approach to the construction of a stocking guide is consistent with the definitions of TAR and CCF. We used relative stand density (RSD, the ratio of absolute stand density to a reference level based on no competition) in the determination of the line of Maximum Average Density (Ernst and Knapp 1985). Maximum crown area (MCA) was calculated by applying the crown radius prediction equation for open-grown trees to each forest-grown tree in a plot and dividing the calculated crown area by the plot area. RSDs were determined for each plot having mortality to provide an index of Average Maximum Stocking. RSDs ranged from 1.3 to 2.9 with an average of 1.98 (Table 1), or 2 for practical purposes. This is consistent with the maximum CCF ceiling of 200 reported by Krajicek et al. (1961). An A-line based on RSD was calculated by multiplying stem density (trees  $ac^{-1}$ ) and basal area for the B-line by 2 (Figure 1).

A maximum reference level of stocking that parallels a minimum reference stocking level is not biologically realistic, particularly in the context of tree mortality. In recent years, the relative density index (Reineke 1933) has been modified to develop relative density guides for tree species across North America (e.g., Jack and Long 1996, Wilson et al. 1999, Saunders and Puettmann 2000) based on the logarithmic relationship between stand diameter and trees  $ac^{-1}$  for undisturbed, fully stocked stands.

Based on the assumption that tree mortality results from self-thinning, the  $-3/2$  power rule, or Yoda's law (Weller 1997) was invoked to construct a reference level for Average Maximum Density independent of the B-line.

**Table 1. Average (range in parentheses) attributes from the European larch plots used to construct average maximum density and open-grown tree full site occupancy reference lines.**

Number plots	Age (yr)	Site index (ft) (bh age 20)	Trees $ac^{-1}$	Basal area $ac^{-1}$ ( $ft^2 ac^{-1}$ )	Average $dbh_q$ *(in.)	Total vol ( $ft^3 ac^{-1}$ )	Total vol (cords $ac^{-1}$ )	Relative stand density (unitless)
16	45 (16–60)	52 (41–60)	357 (142–759)	185 (97–296)	11.0 (5.1–16.6)	6,700 (1,980–13,000)	57 (17–111)	1.98 (1.4–2.9)

\*  $dbh_q$  = quadratic dbh.

**Table 2. Parameter estimates (standard error in parentheses) and fit statistics for the model:  $\ln(Y) = \beta_0 + \beta_1 \ln(X)$  where,  $Y$  is basal area or volume tree<sup>-1</sup>, and  $X$  is number of trees ac<sup>-1</sup>.**

Dependent variable	$\beta_0$ (SE)	$\beta_1$ (SE)	Model MSE	$R^2$	Lilliefors $P$ -value	Bias	Log bias correction factor
$\ln(\text{BA tree}^{-1})$	7.088 (0.8314)	-1.337 (0.1445)	0.101	0.62	0.23	0.015	1.005
$\ln(\text{volume tree}^{-1})$	13.072 (1.230)	-1.774 (0.2138)	0.221	0.47	0.23	1.038	1.025

The equation

$$\ln(Y) = \beta_0 + \beta_1 \ln(X) + \ln(\epsilon), \quad (1)$$

where  $\beta_0$  and  $\beta_1$  are parameter estimates,  $Y$  is basal area tree<sup>-1</sup> or volume tree<sup>-1</sup>,  $X$  is number of stems per ac, and  $\epsilon$  is error assumed to have a log-normal distribution, was fit to the stand level data where self-thinning was assumed to be occurring. Results transformed to their original units were corrected for logarithmic bias (Baskerville 1972). Normality of residuals was evaluated with a Lilliefors test (Zar 1984). Coefficients of determination were calculated from results transformed to their original units of measure (Kvålseth 1985). Bias [(observed - predicted)/ $n$ ] is reported for units transformed to their original units of measure. All analyses were done with SYSTAT (SPSS, Inc. 2000).

## Results

The  $\beta_1$  coefficient for both the prediction of average stem basal area and average stem volume did not differ significantly from the theoretical  $-3/2$  coefficient on which the self-thinning rule is based (Table 2). Equation (1) explained 62% of the variation in basal area and 47% of the variation in volume for fully stocked stands. The hypothesis of normally distributed residuals was not rejected at a probability level of 0.23 for both basal area and volume. Equation (1) under predicted basal area (bias = 0.015) and volume (bias = 1.038) as indicated by average bias. Logarithmic bias correction factors were small for basal area (1.005) and volume (1.025). The paucity of data precludes the validation of Yoda's law for European larch. Our results, however, represent a point of departure for further refinement. Reference points and the

associated equations for the construction of the stocking charts (Figures 1 and 2) are provided in Table 3.

A Maximum Average Density line that is based on the self-thinning rule has greater biological justification than the anamorphic approach to constructing an A-line illustrated in Figure 1. A stocking guide with an Average Maximum Density line developed using the self-thinning rule for field use is provided in Figure 2. Other studies (Gingrich 1967, Halligan and Nyland 1999) suggest that thinning to a relative density of 60% of maximum density creates stand conditions conducive for optimal individual tree growth. The B-line was calculated independently of the line of Average Maximum Density. The 60% Relative Density was calculated by taking 60% of the basal area calculated using equation (1)

## Discussion

The two approaches for calculating an A-line or line for Average Maximum Density are illustrated in Figure 1. Although our approach to the construction of an A-line differs from established norms, results are consistent with the maximum CCF ceiling of 200 reported by Krajicek et al. (1961). However, as Krajicek et al. (1961) indicated in their original work, a CCF ceiling should be thoroughly validated for individual species. This A-line was based on stand density relative to the maximum crown area of open-grown trees (RSD) and has parallel minimum and maximum (B- and A-line) lines (Figure 1). This parallel relationship between lower and upper stocking levels lacks solid theoretical or biological justification. Therefore we invoked, the self-thinning rule also known as Yoda's law to construct an Average Maximum Density line (Figures 1 and 2).

**Table 3. Maximum projected crown area (PCA), stem density, and basal area (BA) at reference levels in the stocking chart developed for European larch.**

Dbh (in.)	PCA* (ft <sup>2</sup> )	B-line <sup>†</sup>		Self-thin line <sup>††</sup>		60% Self-thin line <sup>§</sup>		A-line <sup>  </sup>	
		Trees ac <sup>-1</sup>	BA (ft <sup>2</sup> ac <sup>-1</sup> )	Trees ac <sup>-1</sup>	BA (ft <sup>2</sup> ac <sup>-1</sup> )	Trees ac <sup>-1</sup>	BA (ft <sup>2</sup> ac <sup>-1</sup> )	Trees ac <sup>-1</sup>	BA (ft <sup>2</sup> ac <sup>-1</sup> )
5	88	495	67	800	125	550	75	990	134
6	116	376	74	600	138	422	83	752	148
7	147	296	79	500	147	330	88	592	159
8	182	239	84	450	152	256	89	478	167
9	220	198	87	375	162	220	97	396	174
10	262	166	91	300	175	193	105	332	181
12	358	122	96	250	186	142	112	244	191
14	468	93	100	200	201	113	121	186	199
16	593	73	103	150	222	95	133	146	205
18	732	60	105	130	233	79	140	120	210
20	887	49	107	110	247	68	148	98	214

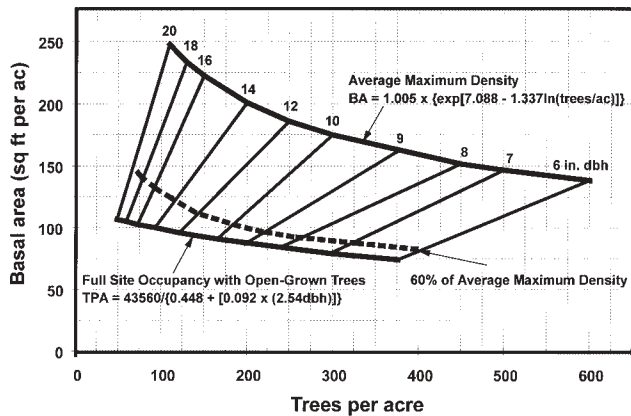
\* PCA =  $\pi \times (3.2808 \times \{0.448 + [0.092 \times (2.54 \times \text{dbh})]\})^2$  [variables (dbh and feet) in the equation for open-grown European larch from Gilmore (2001) was converted from inches to cm and then from meters to feet in PCA calculation].

† B-line trees ac<sup>-1</sup> = 43,560/PCA; B-line BA =  $(0.005454 \times \text{dbh}^2) \times \text{trees ac}^{-1}$ .

†† Self-thin line BA =  $1.005 \times \exp[7.088 \times 1.337 \times \ln(\text{trees ac}^{-1})]$ .

§ 60% self-thin line BA trees ac<sup>-1</sup> =  $(0.6 \times \text{self-thin line BA}) / (0.005454 \times \text{dbh}^2)$ .

|| A-line trees ac<sup>-1</sup> =  $2 \times \text{B-line trees ac}^{-1}$ ; A-line BA =  $2 \times \text{B-line BA}$ .



**Figure 2. Stocking chart for European larch with independent reference lines based on the self-thinning rule and full site occupancy for open-grown trees.**

Yoda's Law suggests a greater capacity for basal area at lower stem densities and a lower capacity for basal area at higher stem densities than the A-line. In other words, greater mortality is predicted at higher densities for smaller diameter trees and less at lower densities for larger diameter trees using the self-thinning line as opposed to the A-line derived from RSD. Data points for stands having tree mortality in the understory (solid circles) are above and below the A-line and self-thinning line. All but one of the data points from stands where self-thinning has not yet occurred (open circles) are below the calculated self-thinning line (Figure 1).

It is assumed that site quality affects the rate of growth but not the relationship between basal area and stems  $ac^{-1}$  to describe stocking. This assumption, however, has not been validated (Leak 1981). Berguson et al. (1994) proposed a relative stocking index and also calculated their line of Average Maximum Density based on the log-log relationship between basal area and number of stems  $ac^{-1}$ . Using data for jack pine, red pine, balsam fir, mesic hardwoods, and aspen they found no relationship between site index and the slope of the log-log equation. They did, however, find a statistical relationship between their proposed index and indirect measures of site quality (temperature, precipitation, silt content of surface soil, annual water deficit). Indirect measures of site quality affect larch

**Table 4. Cubic foot volumes  $ac^{-1}$  at various stand diameters at the average maximum density and full site occupancy reference lines.**

Dbh (in.)	Average max density self-thinning line* ( $ft^3 ac^{-1}$ )	Full site occupancy (B-Line) <sup>†</sup>
5	2,761	718
6	3,485	794
7	4,039	849
8	4,211	903
9	5,099	937
10	6,108	982
12	7,079	1,038
14	8,481	1,084
16	10,705	1,123
18	12,020	1,148
20	13,760	1,174

\*  $V = 1.025 \times \{ \exp[13.072 - 1.744 \ln(\text{dbh})] \} \times \text{trees } ac^{-1}$  (Table 2).

<sup>†</sup>  $V = 8.225(\text{BA}) + 2.306(\text{BA} \times \text{dbh})$  (Gilmore and Briggs 1996).

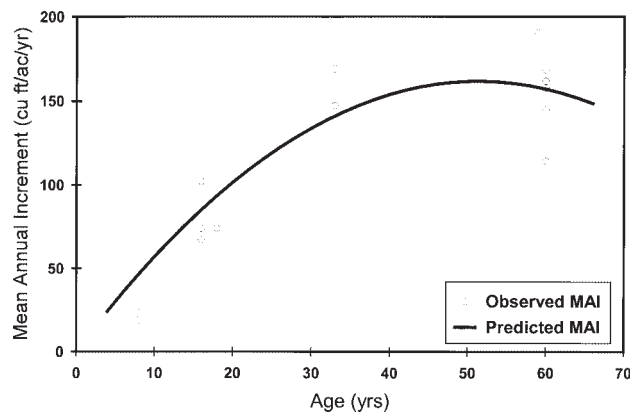
productivity (Gilmore et al. 1994), but we are unable to test the applicability of the technique used by Berguson et al. (1994) to European larch because of the few number of plots (16) having naturally occurring mortality.

An additional assumption implied in stocking guides that requires greater scrutiny is the constant relationship between minimum stocking levels and stand height. This point was originally raised by Wilson (1946). Seymour and Smith (1987) eventually provided quantitative evidence that the B-line for eastern white pine is height dependent. Further research is needed on the validity of a constant crown width-dbh relationship for other species.

## Application

One of the criticisms levied against stocking guides is that they lack a temporal dimension and cannot be incorporated into harvest scheduling models. Nonetheless, given the absence of long-term growth and yield studies for European larch in North America, this stocking chart provides field foresters with a valuable tool for making stand-level decisions on when to thin larch plantations based on stand density and average stand diameter. We attempt to alleviate this lack of a temporal dimension concern by providing additional volume information for resource managers. Volumes ( $ft^3 ac^{-1}$ ) at the Average Maximum Density and Full Site Occupancy reference lines are provided for each diameter class (Table 4). Volumes that could be removed at diameters most likely to be thinned (6 to 10 in. dbh) from the Average Maximum Density to Full Site Occupancy reference lines ranges between  $2600 ft^3 ac^{-1}$  and  $5100 ft^3 ac^{-1}$  (3 and 6 cords  $ac^{-1}$  at  $85 ft^3 cord^{-1}$ ) assuming a geometric thinning and no change in average stem diameter.

To provide additional direction in the development of a temporal dimension to a thinning schedule, we provide an equation to predict mean annual increment (MAI) for the medium quality site category used in an earlier study (Gilmore et al. 1994). Fifteen plots with site indices (index 20 yr bh age) ranging from 54 to 58 were used in the construction of a no-intercept quadratic model to predict MAI from age (Figure 3). Peak MAI for larch growing on medium quality sites, as determined from site index, in Maine is  $162 ft^3 ac^{-1} yr^{-1}$  at



**Figure 3. Observed and predicted mean annual increment (MAI) for medium quality larch plantations in Maine using the quadratic equation:  $MAI = 6.302(\text{age}) - 0.06134(\text{age}^2)$ .**

age 51. Limitations in the size of our data set preclude MAI prediction for high or low quality sites.

The development of thinning schedules would depend on the desired final product and initial planting densities. Low density plantings would reduce the need for thinning but likely result in a greater need for competition control from undesired shrub or tree species until full crown closure occurs. Higher density planting would result in more rapid crown closure, and a thinning likely would be necessary to realize maximum growth potential on individual trees. Volumes removed during thinnings could be part of a strategy to alleviate projected fiber shortages.

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